INVESTIGATION OF TURBULENT PULSATIONS OF THE REFRACTIVE INDEX OF WATER USING A TÖPLER INSTRUMENT

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UDC 532.507

Töpler instruments are finding ever wider application for the investigation of small-scale turbulence. In this case, both photoelectric [1] and photographic [2] recording is being used. The present article describes the use of a towed Töpler instrument with photoelectric recording for investigating the turbulent fluctuations of the refractive index of sea water. The article gives evaluations of the applicability of the instrument, and an example of the measurement results obtained.

Let us examine the schematic diagram given in Fig. 1, which illustrates the operation of a Töpler instrument with photoelectric recording.

An almost parallel beam of light, with a diameter d passes over the path L through the medium under investigation, located between windows 1 and 2, falls on the objective 3, and is concentrated by the objective in the focal plane. The value of the radius of the beam of light in the focal plane r_0 (the radius of the spot), in the absence of optical inhomogeneities in the medium under investigation, is determined by the dispersion of the beam, by its diffraction in the collimating diaphragm, and by the aberrations of the optical system. For a laser Töpler instrument, the contributions made by the first two sources are approximately identical, and are one order of magnitude less than the contribution made by the third source.

In the focal plane, there is a flat Foucault blade 4, behind which is located the photoreceiver 5.

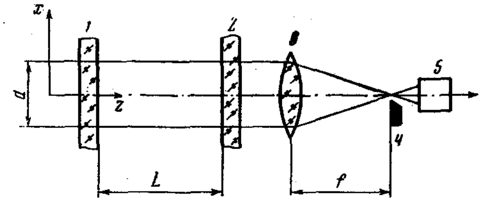

Fig. 1

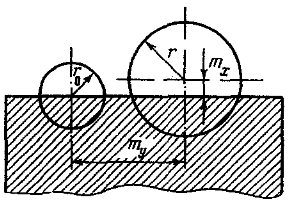

Fig. 2

The Foucault blade is an opaque diaphragm, which cuts off part of the light on the path to the photoreceiver. The position of the blade is so chosen that, in the absence of fluctuations of the refractive index in the medium under investigation, the position of the edge of the blade will coincide with the diameter of the spot in the focal plane.

The presence of fluctuations in the refractive index breaks down the original parallel character of the rays, and leads to an increase in the spot up to a dimension equal to r, and to a shift in its original position (Fig. 2).

The increase of the spot takes place as a result of scattering of the light in the fluctuations of the refractive index, whose linear dimension is small in comparison with the value of the path of the beam L; the shift of the spot as a whole is determined by deflection of the beams in fluctuations of the refractive index, whose dimension is equal to or greater than the length of the path L.

The shifts of the spot along the x and y axes are equal to

Moscow. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 5, pp. 103-108, September-October, 1971. Original article submitted May 3, 1971.

$$m_i = f\theta_i = \frac{f}{n_0}\int_0^L \frac{\partial n}{\partial x_i}\,dz \tag{1}$$

Here i = x, y; θ is the angle of deflection of the beam of light at the outlet from the region under study; f is the focal distance of the objective; n_0 is the refractive index of air.

The axially symmetrical increase of the spot resulting from scattering due to small fluctuations of the refractive index, is determined by the relationship [3]

$$\langle r^2\rangle = f^2\langle\theta^2\rangle = \frac{2}{3}f^2\langle(n-n_*)^2\rangle\frac{L}{\lambda_0} \tag{2}$$

Here $\langle\theta^2\rangle$ is the variance of the angle of scattering of the beams at the outlet from the volume under consideration; $n - n_*$ is the turbulent pulsation of the refractive index; $n_* = \langle n\rangle$.

Relationship (2) is valid with satisfaction of the approximation of geometric optics ($\lambda \ll \lambda_0$, $\sqrt{\lambda L} \ll \lambda_0$) of the condition $\lambda_0 \ll L$, as well as of the condition of isotropicity and homogeneity of the turbulent field. Here λ is the wave length of the light; λ_0 is the minimal size of the inhomogeneities, for which we may take, for example, the Kolmogorov scale.

The instrument records the total light flux, passing by the blade and falling on the photoreceiver. Therefore (Fig. 2), a shift of the spot along the edge of the blade (a change in m_y) does not lead to a change in the photocurrent. A shift in a direction perpendicular to the edge of the blade leads to a change in the light flux which, for example, with a Gaussian distribution of the intensity of the light in the spot, is equal to

$$\frac{\Phi}{\Phi_0} = \frac{1}{\pi}\int_0^{\pi/2}\exp\left\{-\frac{m_x^2}{\langle r^2\rangle\sin^2\varphi}\right\}d\varphi \tag{3}$$

Here Φ is the light flux falling on the photoreceiver; Φ_0 is the total light flux in the spot.

With a parabolic distribution of the intensity of the light in the spot we obtain the following relationship:

$$\frac{\Phi}{\Phi_0} = \frac{1}{2} + \frac{1}{\pi}\arcsin\frac{m_x}{\sqrt{\langle r^2\rangle}} + \frac{m_x}{\sqrt{\langle r^2\rangle}}\sqrt{1 - \frac{m_x^2}{\sqrt{\langle r^2\rangle}}}\left(\frac{5}{3} - \frac{2m_x^2}{3\langle r^2\rangle}\right) \tag{4}$$

As is evident from these relationships, a change in the value of $\langle r^2\rangle$ without a shift in the center of the spot does not lead to a change in the photocurrent, if the blade divides the spot in half.

To record changes in the variance of the scattering angles, we must use the dark-field system (the blade overlaps a large part of the spot).

With small values of the shift, m_x, in relationships (3) and (4), we can limit ourselves to the linear terms of their expansion in a Taylor series. Here, the relationship $\Phi = \Phi(m_x)$ is identical in both cases and is close to linear. This relation may be used to determine and calculate the instrument constant, determined as

$$A = du/d\theta_x$$

where u is the voltage of the effective signal in the load on the photomultiplier.

Calculation gives the following expression for A:

$$A = \frac{2\gamma f}{\pi r}\Phi_0$$

where $\gamma = \partial u/\partial\Phi$ is the sensitivity of the photoreceiver with respect to the voltage.

The instrument used in the present work was calibrated; as a result, a value of 10^5 V was obtained for A.

Thus, in an analysis of the operation of a Töpler instrument, two regions may be separated out

$$\langle r^2\rangle = \text{const}, \qquad m_x = \text{var}$$
$$\langle r^2\rangle = \text{var}, \qquad m_x = \text{const}$$

The first is valid when Λ, i.e., the linear scale of the turbulent pulsations of the refractive index, is equal to or greater than the base of the change L; the second is valid when the scale of the pulsations is much less than L, but greater than the diameter of the beam.

With operation of the instrument in these regions, analysis of the measurement results is comparatively simple.

Fluctuations of the refractive index, whose linear dimension is on the order of d, leads to a non-axially symmetrical deformation of the spot; fluctuations with a linear dimension less than d lead to a redistribution of the light in the spot, proportional to the second derivative of the refractive index, and are not recorded by an instrument of this type.

With $L \leqslant \Lambda$, we can find the correlation function for the field of the refractive index of the medium from the correlation function of the measured signal, if the hypothesis with respect to the frozen character of the turbulence is satisfied.

With $L \gg \Lambda$ under the assumption of local isotropicity and of homogeneity of the turbulent pulsations of the refractive index, we can find the variance of the refractive index and, with a given type of spectrum, we can determine the structural constant of the medium. Thus, for example, for the inertial range, when the spectral density of the field of the pulsations of the refractive index is equal to [4]

$$F_n(k) = 0.033 C_n^2 k^{-11/3} \exp\{-0.16 k^2 \lambda_0^2\} \tag{5}$$

for the structural constant of the medium, C_n^2, we have

$$C_n^2 = 1.22 \langle r^2 \rangle \lambda_0^{1/3} / f^2 L \tag{6}$$

where k is the wave number.

In the remaining cases, when there is simultaneous variation both of the radius of the spot and of its shift, analysis of the measurement results is considerably more complex.

Returning to the region $L \leqslant \Lambda$, we evaluate the connection between the correlation functions of the field of the refractive index B_n and the field of the recorded signal B_u, under the assumption of the smallness of the shift of the spot. In this case, the fluctuation component of the signal is analyzed.

In this case, there is a linear connection between the recorded signal and the angle of deflection of the beam at the outlet from the region under study. Then, taking account of (4), we have

$$B_u(x_1, x_2, t_1, t_2) = \frac{A^2}{n_0^2} \int_0^L \int_0^L dz_1\, dz_2 \frac{\partial}{\partial x_1} \frac{\partial}{\partial x_2} B_n(x_1, x_2, 0, z_1, z_2, t_1, t_2) \tag{7}$$

This relationship is valid when the condition $d \ll \lambda_0$ is satisfied, and integration over the diameter of the beam can be neglected.

Assuming that the field of the turbulent fluctuations of the refractive index is stationary, homogeneous, and isotropic, as well as accepting the hypothesis that the field is frozen, relationship (7) can be written in the following manner:

$$B_u(\xi) = -\frac{2LA^2}{n_0^2} \frac{\partial^2}{\partial \xi^2} \int_0^L \left(1 - \frac{\eta}{L}\right) B_n(\xi, 0, \eta)\, d\eta \tag{8}$$

Here

$$\xi = x_1 - x_2, \quad \eta = z_1 - z_2, \quad \rho^2 = \eta^2 + \xi^2, \quad B_n(\xi, \eta) = B_n(\rho)$$

Let us analyze the expression obtained for different ratios of the length of the beam to the scale of the inhomogeneities. We shall consider the three regions $\Lambda > L$, $\Lambda < L$, $\Lambda \sim L$.

In the first region, when the length of the beam is less than the scale of the inhomogeneities, is can be assumed that B_n does not depend on η and that, consequently,

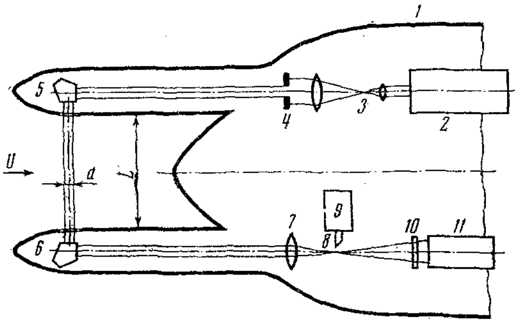

Fig. 3

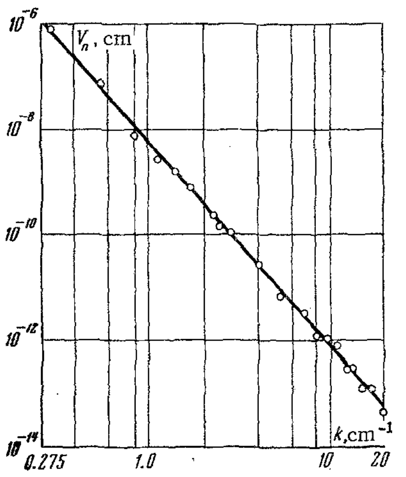

Fig. 4

$$B_u(\xi) = -\frac{L^2A^2}{n_0^2}\frac{\partial^2}{\partial\xi^2}B_n(\xi) \tag{9}$$

Carrying out a one-dimensional Fourier transformation, we obtain a connection between the one-dimensional spatial spectrum of the pulsations of the refractive index $V_n(k)$ and the spectrum of the recorded signal $V_u(k)$

$$V_n(k) = \frac{n_0^2}{A^2L^2}\frac{V_u(k)}{k^2} = b\frac{V_u(k)}{k^2} \tag{10}$$

Here b is the instrument constant.

In the second region, when the length of the beam is greater than the scale of the inhomogeneities, in formula (8) we can neglect the value of η/L in comparison with unity, and can replace the upper integration limit by infinity [4]. In this case we obtain

$$B_u(\xi) = -\frac{2L^2A^2}{n_0^2}\left(\frac{\Lambda}{L}\right)\frac{\partial^2}{\partial\xi^2}B_n(\xi) \tag{11}$$

$$V_n(k) = \frac{b}{2}\left(\frac{L}{\Lambda}\right)\frac{V_u(k)}{k^2} \tag{12}$$

For the third region, simple relationships cannot be obtained, and formula (8) must be used. The integral entering into this formula can be calculated if the form of $B_n(\rho)$ is known. For example, for $B_n(\rho) = a^2\exp\{-\rho^2/\Lambda^2\}$

$$V_n(k) = b_1\left(\frac{L}{\Lambda}\right)\frac{V_u(k)}{k^2}$$

Here

$$b_1 = \frac{b}{\sqrt{\pi}\,\mathrm{erf}\,(L/\Lambda) - [1 - \exp\{-L^2/\Lambda^2\}]\,\Lambda/L}, \quad \mathrm{erf}\,(x) = \frac{2}{\sqrt{\pi}}\int_0^x e^{-t^2}dt$$

The present authors made a study of the turbulent fluctuations of the refractive index of sea water, using an instrument towed from the side of the scientific-research ship "Akademik Kurchatov," during its ninth trip.

Figure 3 shows a schematic diagram of the instrument. The instrument was encased in housing 1. Radiation from a helium–neon laser 2, passing through collimator 3 and collimating diaphragm 4, having an opening with a diameter of 5 mm, through the pentaprism 5 into the working gap, located in the forward part of the housing. To decrease to a maximal degree the effect of the boundary layer at the housing of the instrument on the measurement results, using a fairing, the working gap was carried out to a distance of 120 mm from the nose of the housing.

At the outlet from the working gap L, with a length of 100 mm, the beam is deflected by pentaprism 6 and is focused by objective 7. In the focal plane of the objective there is a Foucault blade 8, whose edge is perpendicular to the axis of the beam. The positions of the Foucault blade are regulated by the spacing mechanism 9. After the Foucault blade there are arranged the interference filter 10 and the photomultiplier 11. Towing of the instrument was done using a towing system [5].

The spectrum of the signals recorded was found using an S5-3 spectroanalyzer, with an analysis band of 6 Hz, and an averaging time of 30 sec.

As an example, Fig. 4 gives the results of measurements, in the form of the spectrum of the fluctuations of the refractive index, obtained with towing of the instrument at a depth of 30 m at a velocity of 2.22 m/sec in a tropical region of the Atlantic Ocean. The scale is logarithmic along both axes.

The above spectrum was obtained by recalculation of the spectrum of the recorded signals, using the relationships given above. The straight line, given on the figure, corresponds to $k^{-3.9}$. The upper limit of the recorded wave numbers was 25 cm^{-1}, which corresponds to a dimension of the optical inhomogeneities on the order of half the diameter of the light beam in the medium under study. The form of the above spectrum is in agreement with experimentally found spectra, under various marine conditions, for example [6].

Since the refractive index of water is a function of temperature, salinity, and pressure, its fluctuations are given by the fluctuations of the above parameters.

The contribution made by the fluctuations of the temperature, salinity, and pressure may be different under different hydrological conditions; it can be found if independent means are used to determine the fluctuations of two of these parameters.

For conditions ordinarily encountered in the ocean, the role of pressure fluctuations is small. For the most widely used relationships between the fluctuations of the salinity and the temperature [7], the contributions made by each of these parameters are approximately identical.

LITERATURE CITED

1. V. N. Stasenko, "Use of a shadow instrument for determining the characteristics of turbulence," Zh. Prikl. Mekhan. i Tekh. Fiz., No. 3 (1970).
2. L. L. Thompson and L. S. Taylor, "The investigation of turbulence using schlieren photography," Raketnaya Tekhnika i Kosmonavtika, 7, No. 10 (1969).
3. M. M. Weiner, "Atmospheric turbulence in optical surveillance systems," Appl. Optics, 6, No. 11 (1967).
4. V. I. Tatarskii, Propagation of Waves in a Turbulent Atmosphere [in Russian], Izd. Nauka, Moscow (1967).
5. V. T. Paka and V. I. Dedkov, "The possibility of investigating the small-scale structure of hydrophysical fields in the ocean using towing systems," Izv. Sib. Otd. Akad. Nauk SSSR, Ser. Tekh. Nauk, issue 3, No. 13 (1969).
6. V. V. Efimov and G. N. Khristoforov, "Wave and turbulent components of the velocity spectrum in the upper layer of the ocean," Izv. Akad. Nauk SSSR, Fizika Atmosfery i Okeana, 7, No. 2, 200-211 (1971).
7. H. L. Grant, B. A. Hughes, W. M. Vogel, and A. Moilliet, "The spectrum of temperature fluctuations in turbulent flow," J. Fluid Mech., 34, Part 3, 423-442 (1968).